First Orbit Determination with Numerical Methods based on Short Arcs Acquired in Space Debris Surveys

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Outline

- Introduction
- Method
- Application
- Outlook
- Summary

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Outline

- Introduction
  - Short arcs from optical surveys (angular measurements)
  - Space-based observations
  - Correlation in survey-only strategies

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Motivation

- Surveys for space debris
  - Improvement and validation of space debris environment models
  - Build-up and maintenance of catalogue of orbital elements
- Initial orbit determination (IOD) is a crucial step :
  - Size estimation from apparent magnitude and orbit (if statistically sampling)
  - IOD limits
    - Performance of catalogue correlation
    - Re-acquisition scenario
    - Object identification
  - Combining heterogeneous tracks ('arclets') from multiple sites
  - Measurement accuracy (pixel scale, SNR limits)
- Typical surveys provide very short arclets
  - Limits of the current sensor technology : FoV diameter vs. aperture
  - Survey patterns aiming to cover larger areas

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Method

- Implementation into the `CelMech` program system (Beutler, 2005)
  - `ORBDET` module : circular OD, “boundary value”, improvement step
  - Consideration of angular measurements (optical observation systems)
- New : `BNBN2D` = derivation of the “boundary value” method
  - Two-dimensional search : systematically varying topocentric ranges at boundary epochs of the observed arc
  - Approximation of observed arc through truncated Taylor series
  - Identification of local minima of the “observed-computed” residuals (over all available observations within arc)
  - Consider the particular solution of the equation of motion

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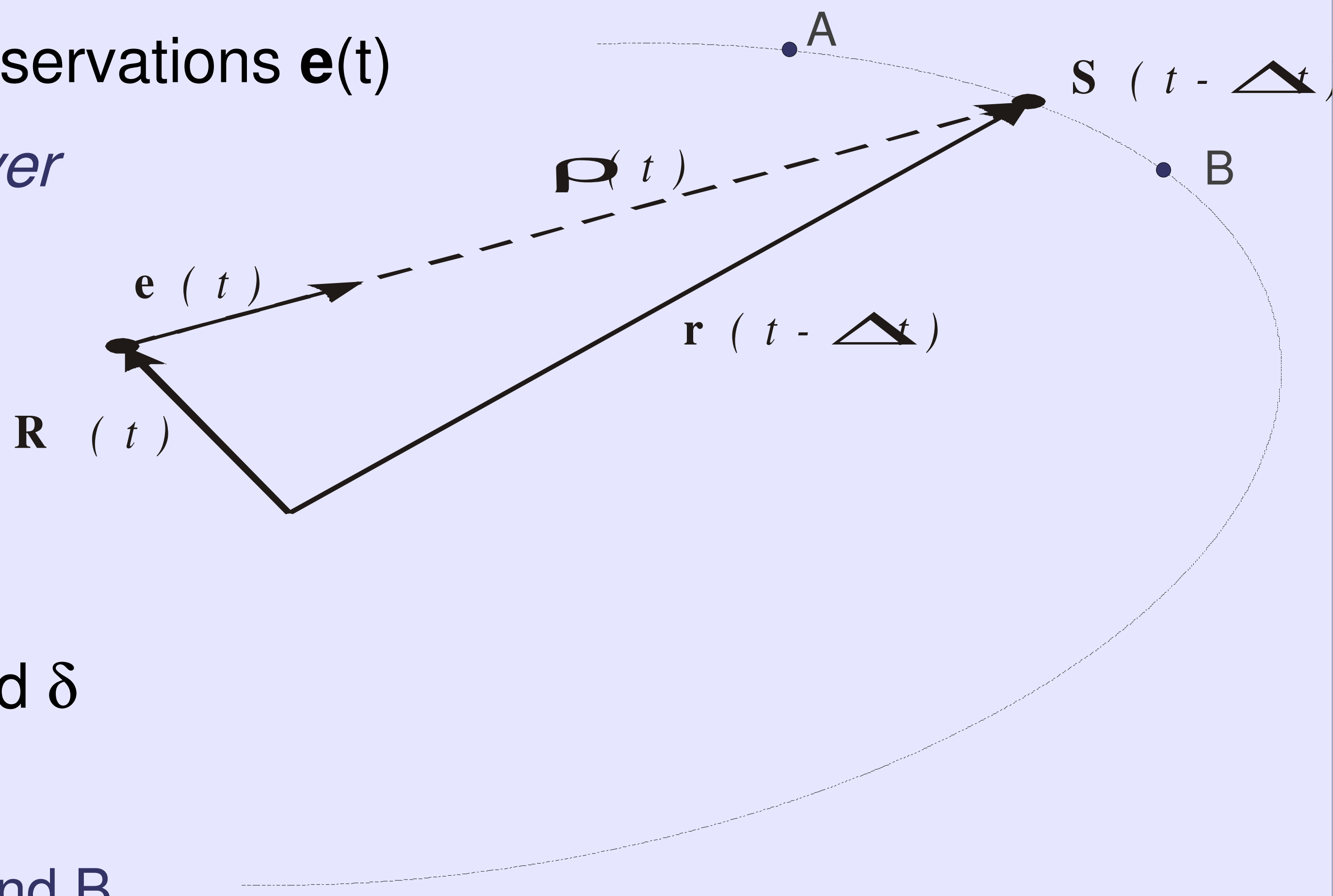
Method

- Observer **R**(t), observations **e**(t)
  → *moving observer*
- Object **S**(t)
- Topocentric ranges **ρ**(t)
- **e**(t) in equatorial coordinates α and δ
- Arclet :
  - Boundaries A and B
  - *n* observations in (A,B)
  - Approximations for A and B from observations



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Method

Taylor-series approximation

$$\mathbf{r}(t) = \sum_{j=0}^{n} (t_A - t)^j \, b_j$$

$$\ddot{\mathbf{r}}(t) = \sum_{j=2}^{n} j(j-1)(t_B - t)^{j-2} \, b_j$$

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$$\mathbf{A} \;\; \mathbf{b} \; = \; \mathbf{l}$$

$$\mathbf{A} = \begin{bmatrix} 1 & t_A - t & (t_A - t)^2 & (t_A - t)^3 \\ 1 & t_B - t & (t_B - t)^2 & (t_B - t)^3 \\ 0 & 0 & 2 & 6(t_A - t) \\ 0 & 0 & 2 & 6(t_B - t) \end{bmatrix}$$

$$\mathbf{b} = (b_0, b_1, b_2, b_3)^T$$

$$\mathbf{l} = \left(r_A, r_B, -GM\frac{r_A}{r_A^3}, -GM\frac{r_B}{r_B^3} \right)^T$$

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Taylor-series approximation

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solve for **b** : **A** is static (and for *n* = 3 $\mathbf{A}^{-1}$ is easy), modify only **l**

for various combinations of ρ_A and ρ_B

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Method

- Search for optimal fit to observation arc
- Criterion : RMS of "observed-computed" of α and δ

$$B\left(\varrho_A,\varrho_B\right)\approx\sqrt{\frac{\sum_{i=1}^{n_{obs}}\cos^2(\delta_i^O)\left(\alpha_i^O-\alpha_i^C\right)^2+\left(\delta_i^O-\delta_i^C\right)^2}{2\,n_{obs}}}$$

- Previous implementation : 1D-search $\rho_B(\rho_A)$ → 2D-search
- Problem : find minima (maybe sharp)
  - Adaptive fine-search
  - Repeat entire estimation with t' replacing t

    (if enough observations are available)
- Example : space-based observation from GEO sensor (t =280s)

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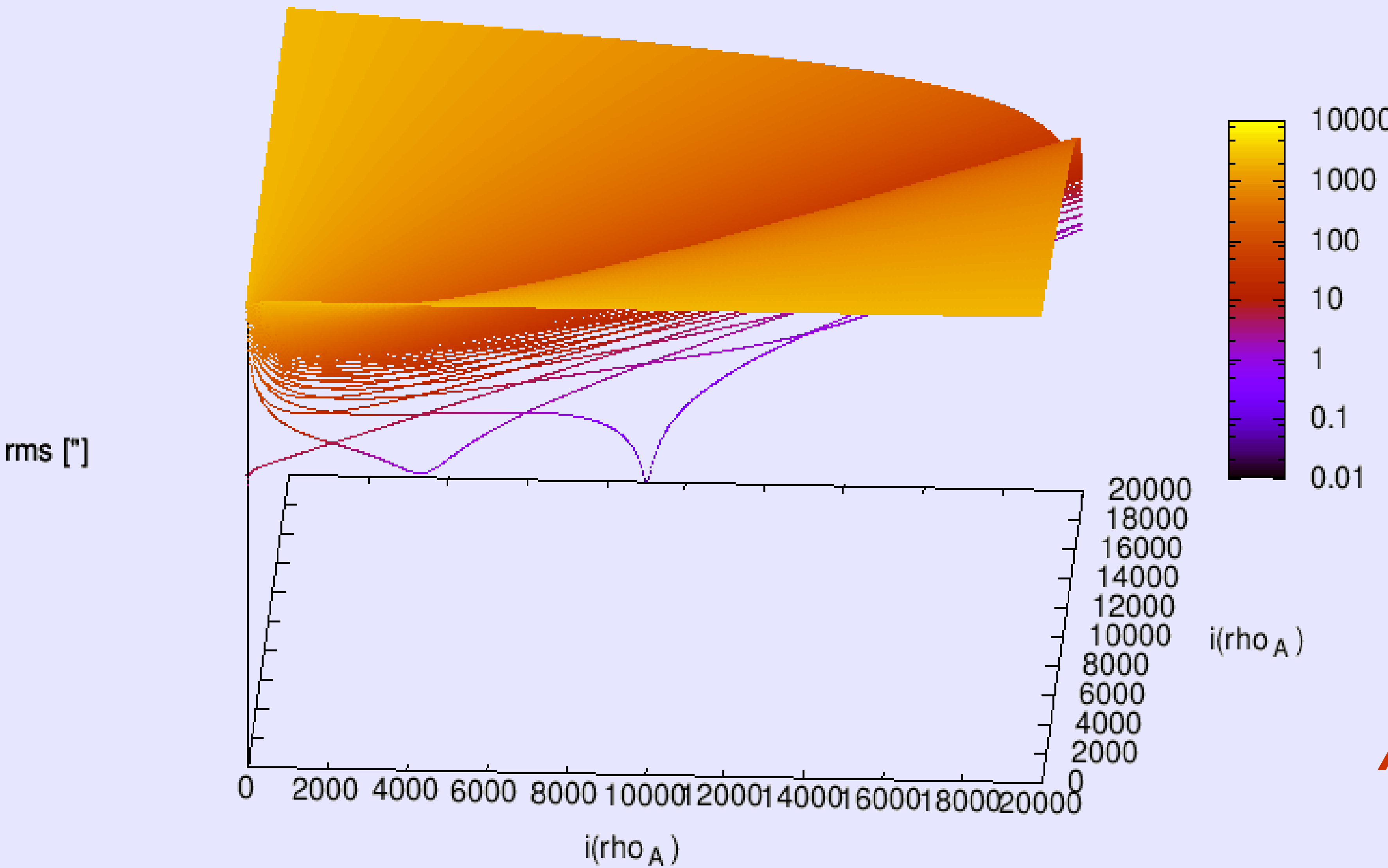
Minima search σ=0.0”



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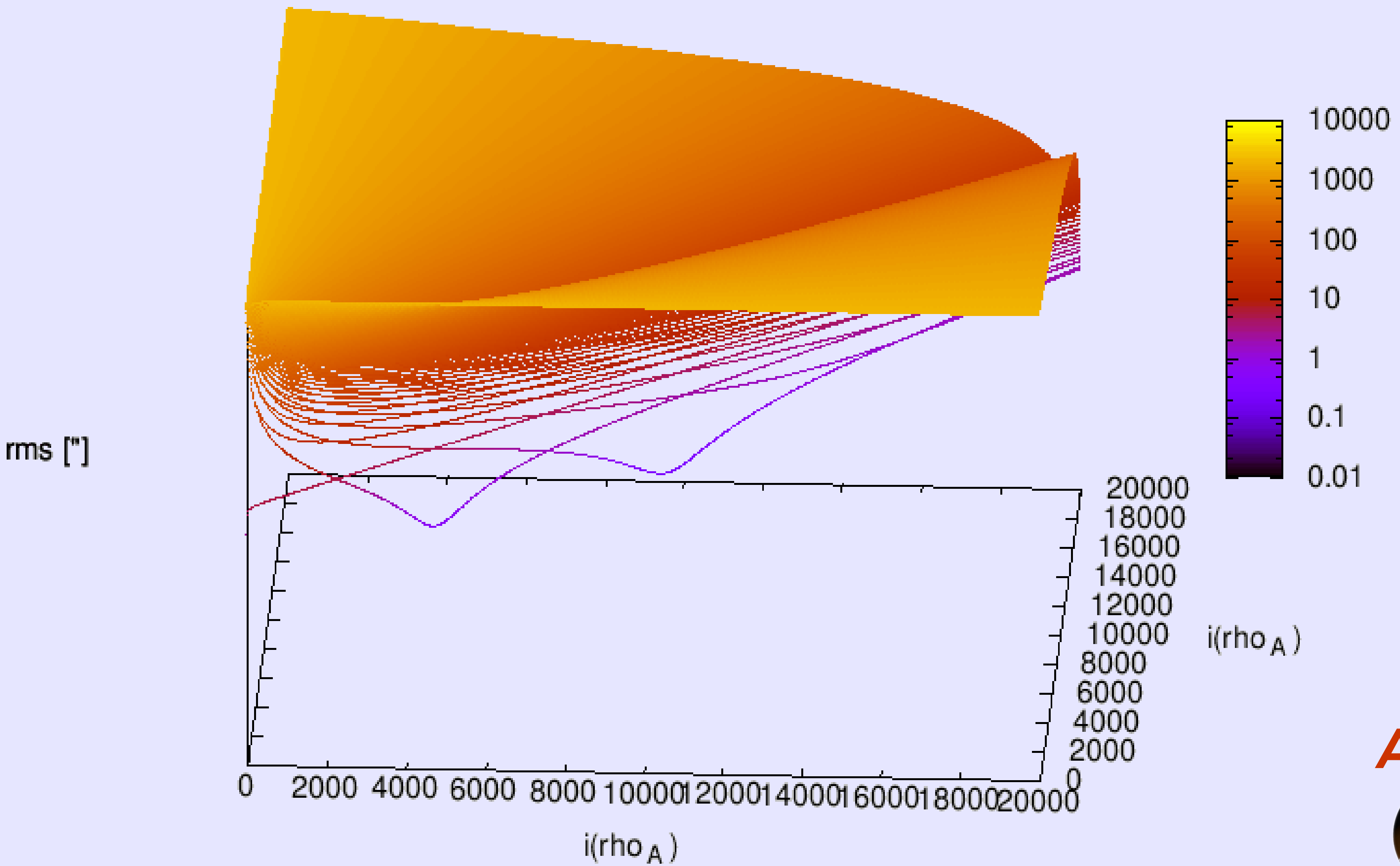
Minima search σ=1.0”



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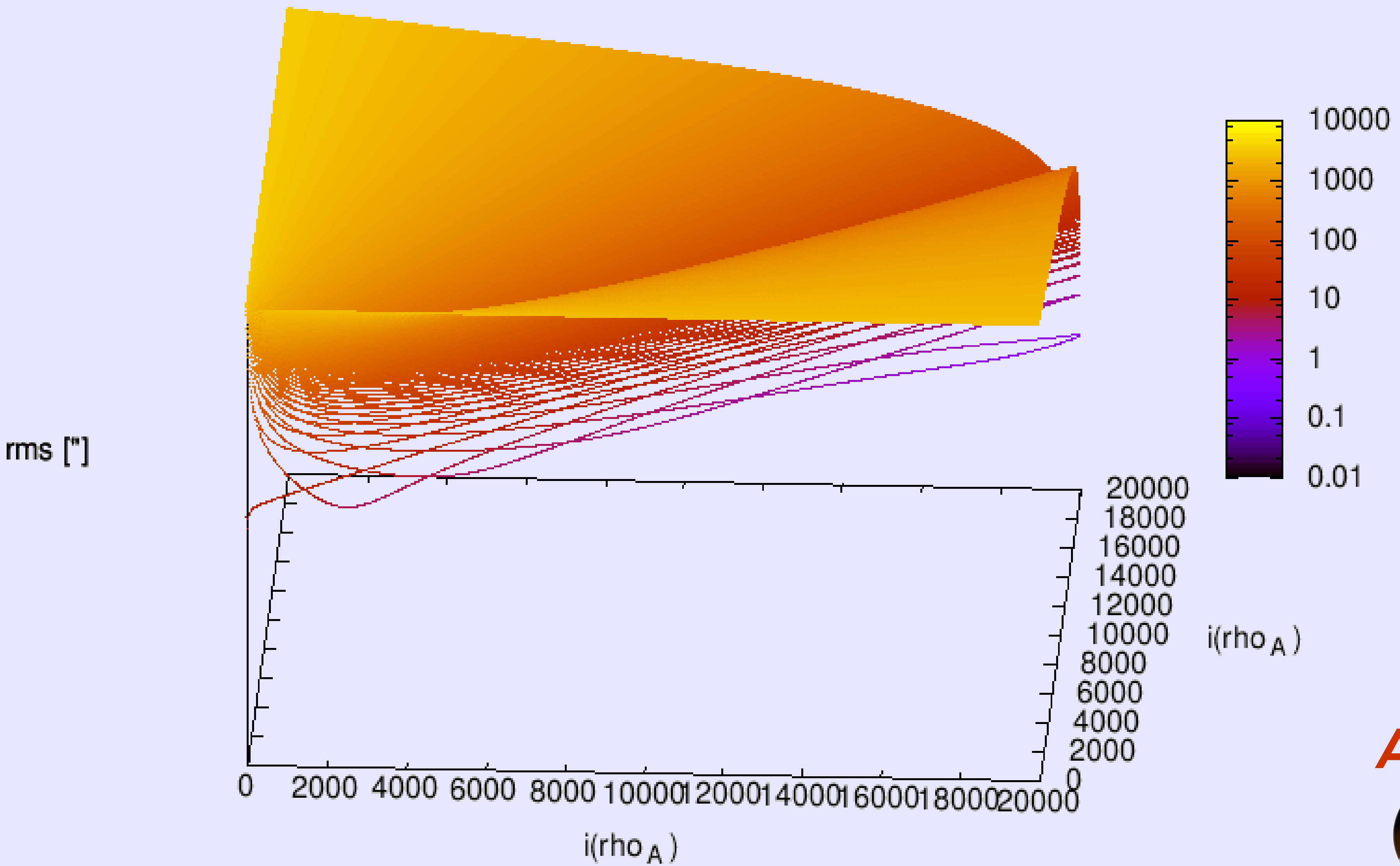
Minima search σ=2.5"



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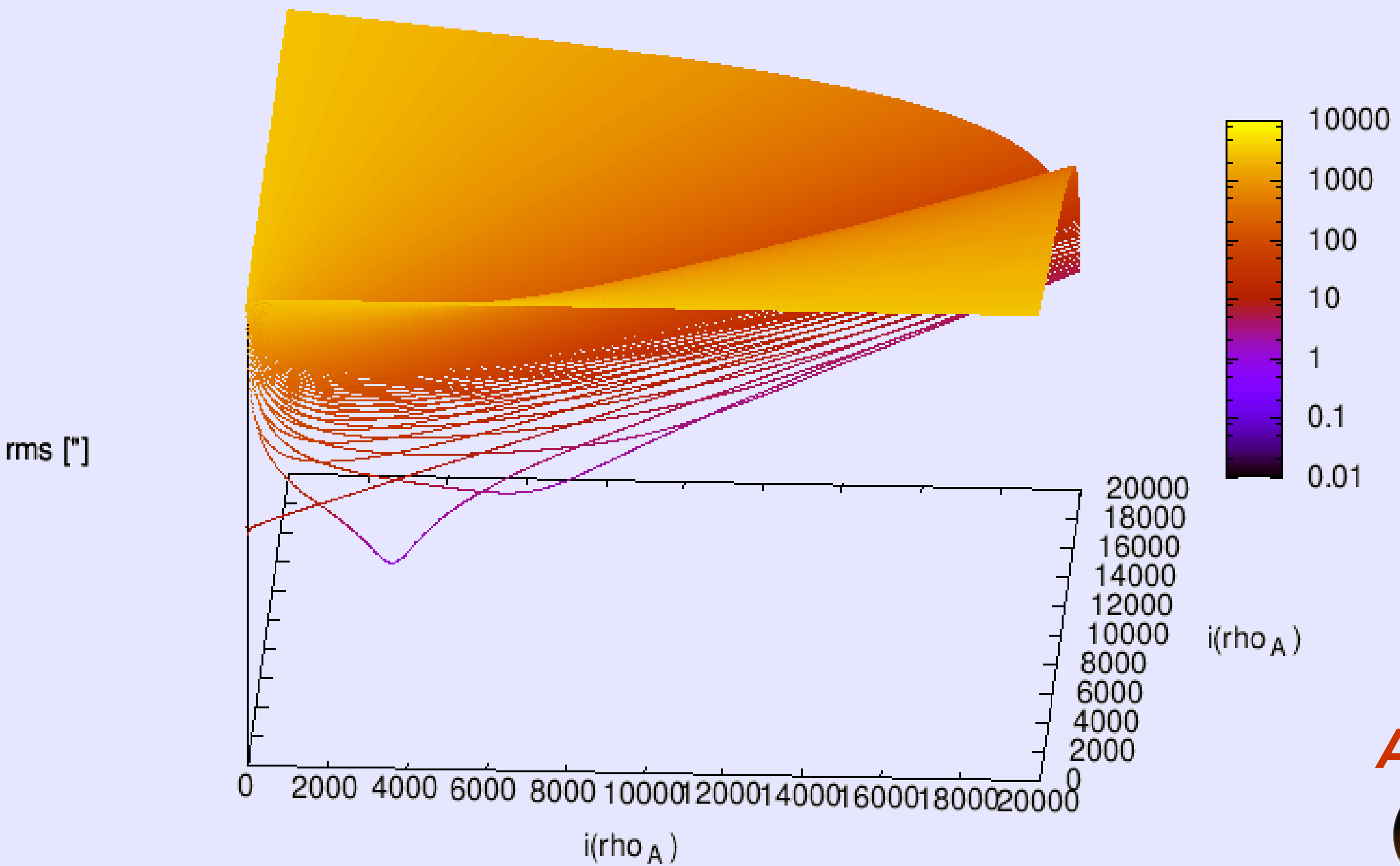
Minima search σ=5.0”



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  - Simulated space-based observations
  - Application to ESA surveys

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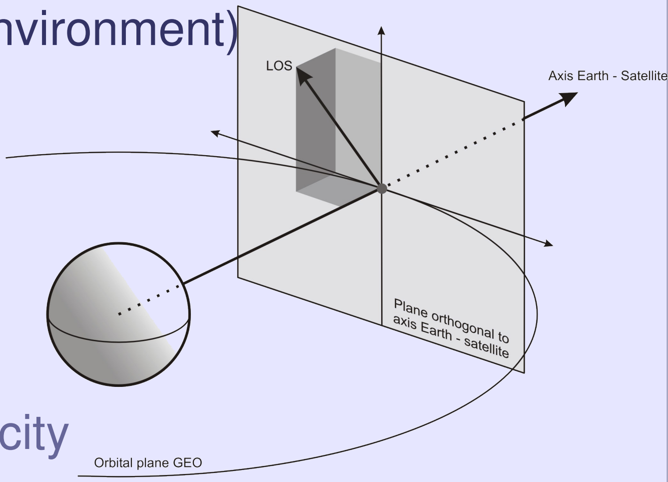
Simulated space-based obs.

- ESA study "Space-Based Optical Observation of Space Debris" : ASRO, NLR, AIUB (goal : characterisation small-sized environment)
  - LEO (SSO) observing LEO
  - subGEO observing GEO
  - GEO (piggy-back, North-pointing)
- Simulation method
  - ESA's PROOF : topocentric ranges and relative velocity
  - ORBDET : simulate observations (with astrometric noise :

    Noise levels (single obs) : 0", 0.5", 1", 2.5", 5")
- Test : Variation of "middle" epoch *t*
- Evaluation of determined orbital elements vs. middle epoch

  → analysis of repeated solutions may partly compensate noise!



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Simulated space-based obs.

Object	a [km]	e [-]	i [deg]	Ω [deg]	arc [s]
GEO-1	*28568*	*0.48*	*13.50*	*96.29*	*282*
GEO-2	26723	0.65	66.94	-101.83	120
GEO-3	*40950*	*0.03*	*11.14*	*42.34*	*212*
GEO-4	24681	0.72	6.83	105.47	210
GEO-5	*34060*	*0.22*	*3.93*	*-23.71*	*776*
LEO-1	7923	0.00	102.38	177.92	24
LEO-2	8093	0.02	74.07	146.40	11
LEO-3	10070	0.32	57.04	-134.30	26
LEO-4	7771	0.01	101.73	150.63	48
LEO-5	25085	0.72	4.24	-81.80	9

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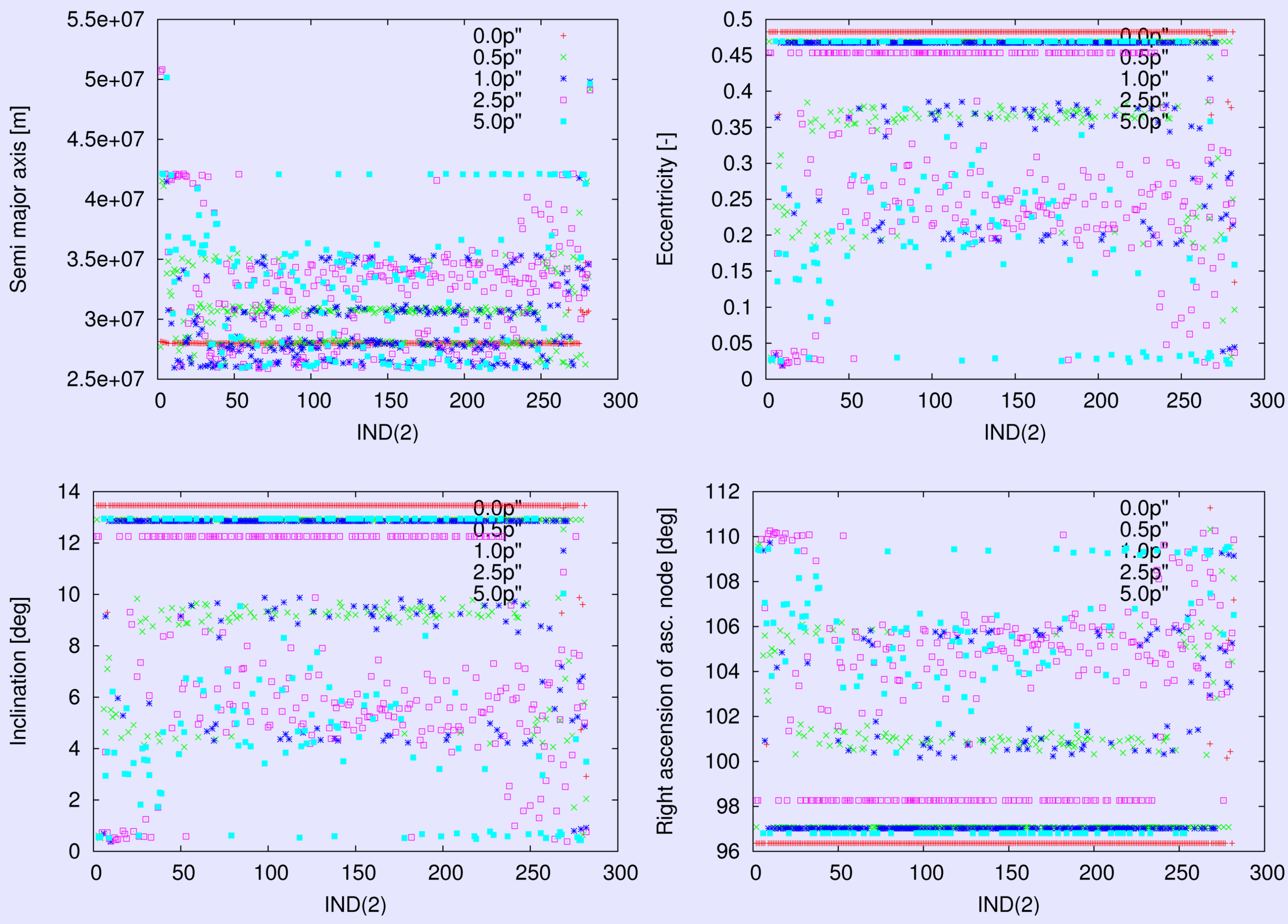
Simulated space-based obs.

GEO-1



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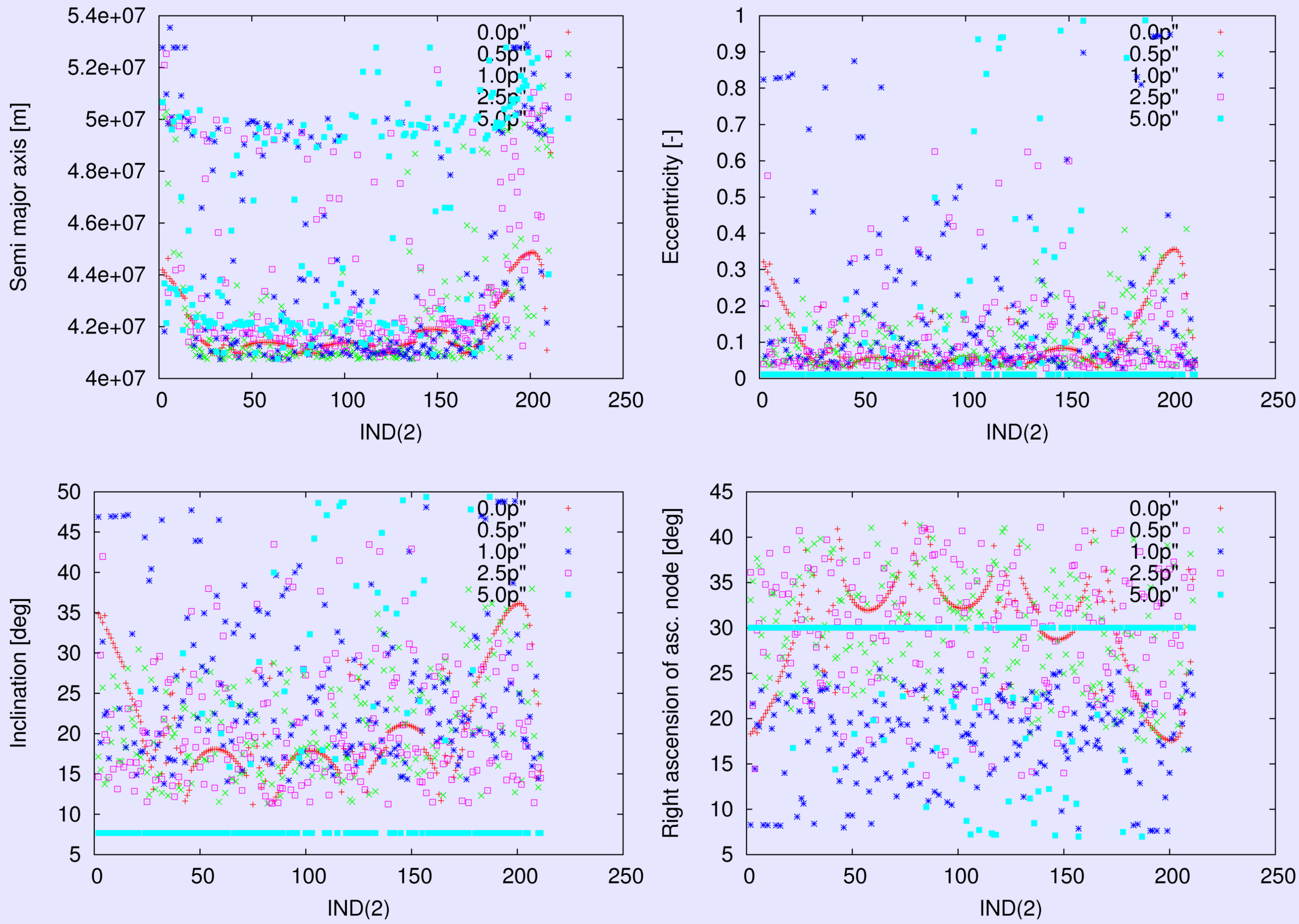
Simulated space-based obs.

GEO-3



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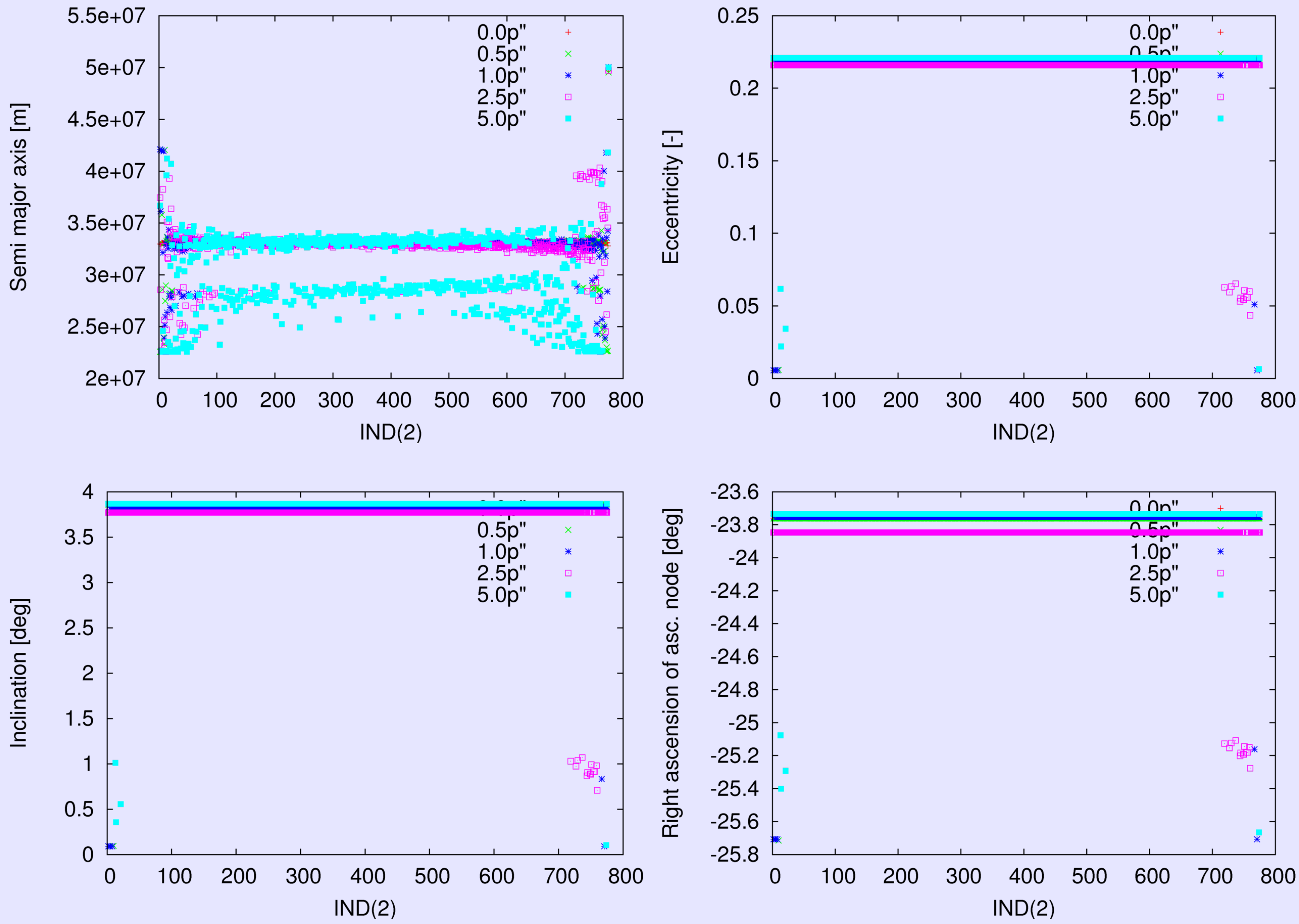
Simulated space-based obs.

GEO-5



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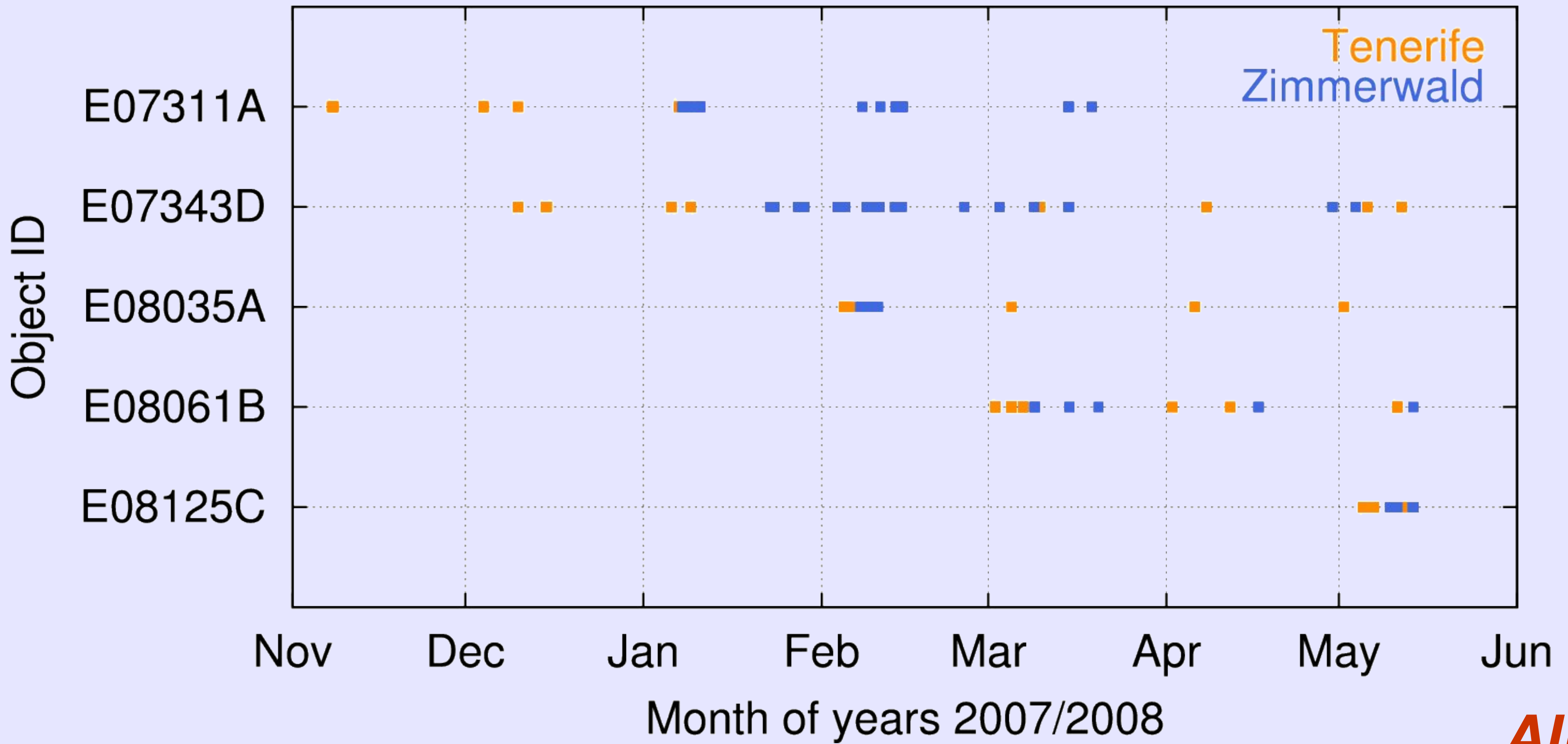
Application to ESA surveys



Application to ESA surveys

Object	a [km]	e [-]	i [deg]	Ω [deg]	Class
E07311A	41334	0	9.52	-32.45	GEO
E07343D	42026	0.49	11.23	3.55	AMR GEO
E08035A	41206	0.05	12.95	-0.14	GEO
E08061B	23162	0.71	7.3	23.23	GTO
E08125C	42302	0.21	6.65	-41.03	AMR GEO

- Results in i - Ω and in e - a space (with errorbars)
  - Faint dot : from single arclet
  - Prominent dot : from two consecutive arclets
  - Indicator "truth" : data from long arc fit (months)
    (circle refers to middle epoch of all observations)

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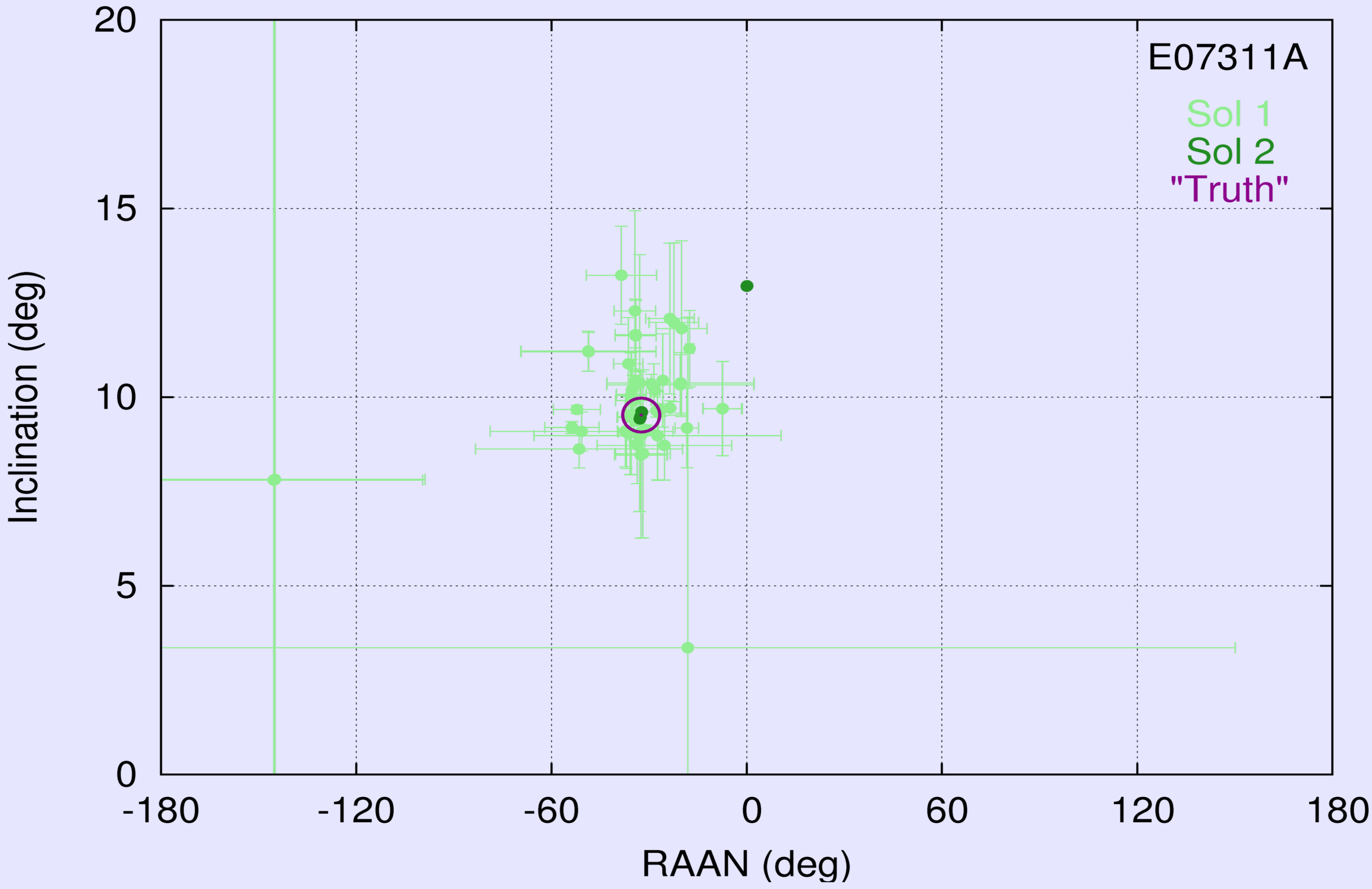
Application to ESA surveys

E07311A



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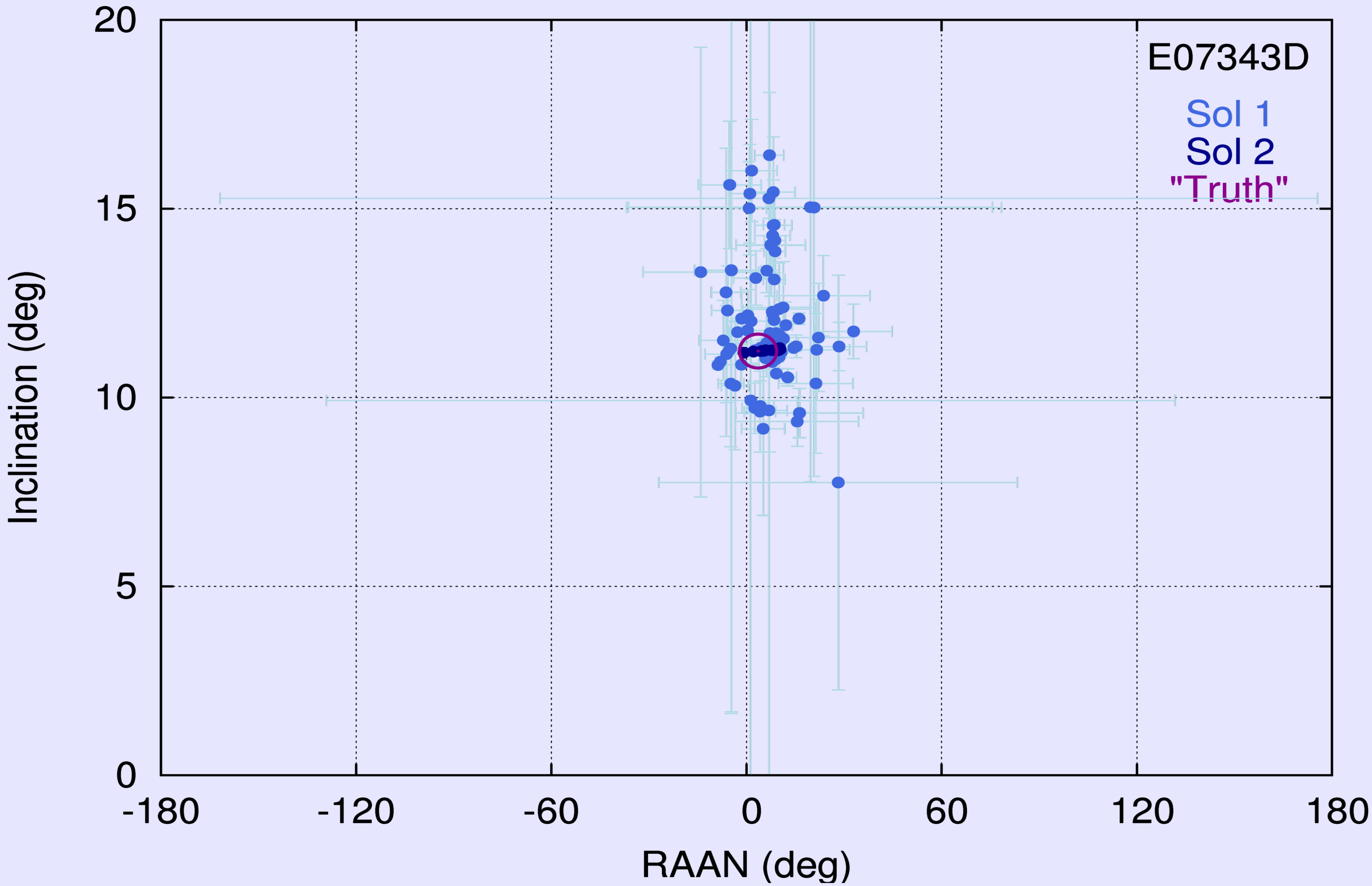
Application to ESA surveys

E07343D



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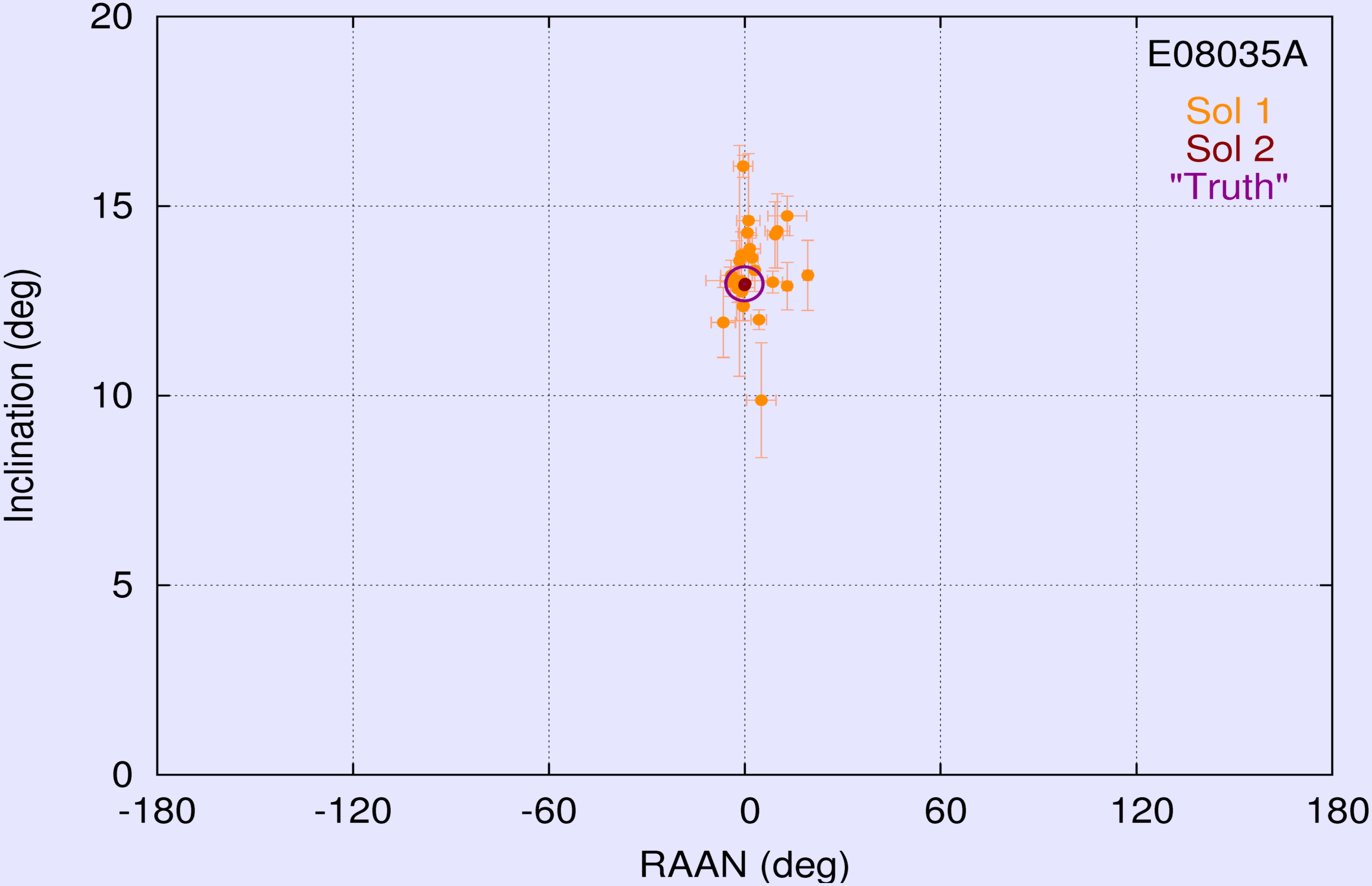
Application to ESA surveys

E08035A



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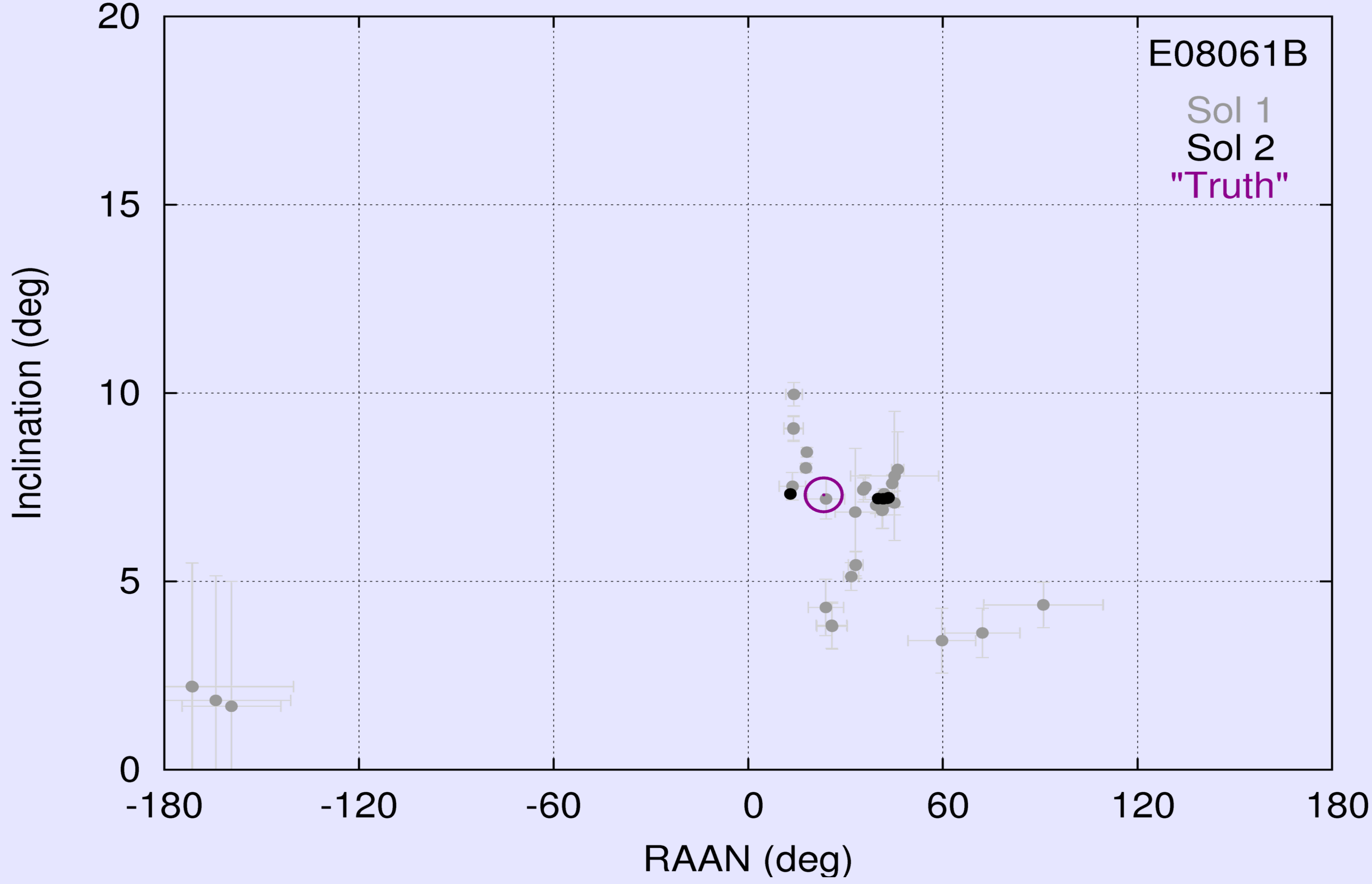
Application to ESA surveys

E08061B



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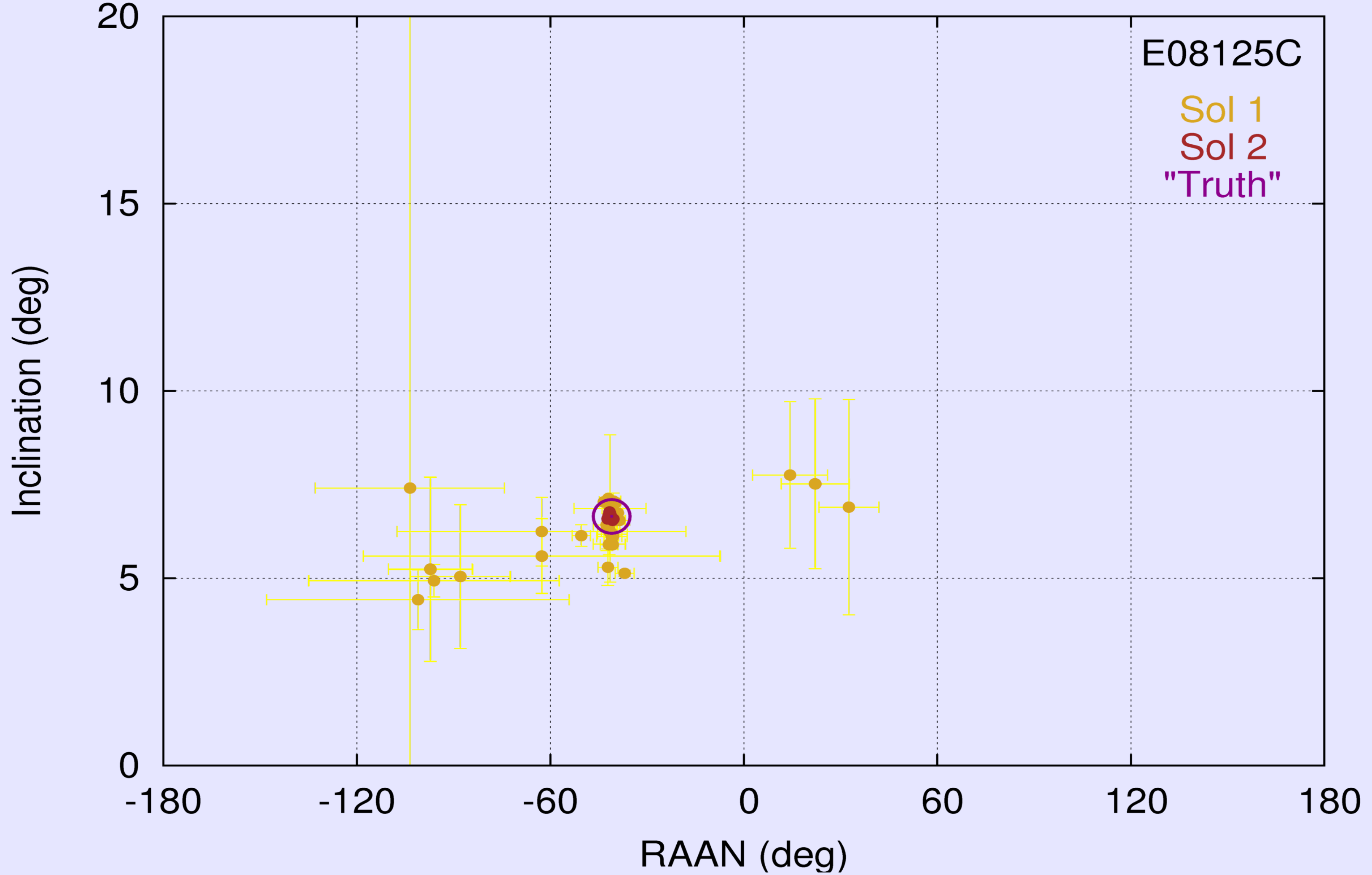
Application to ESA surveys

E08125C



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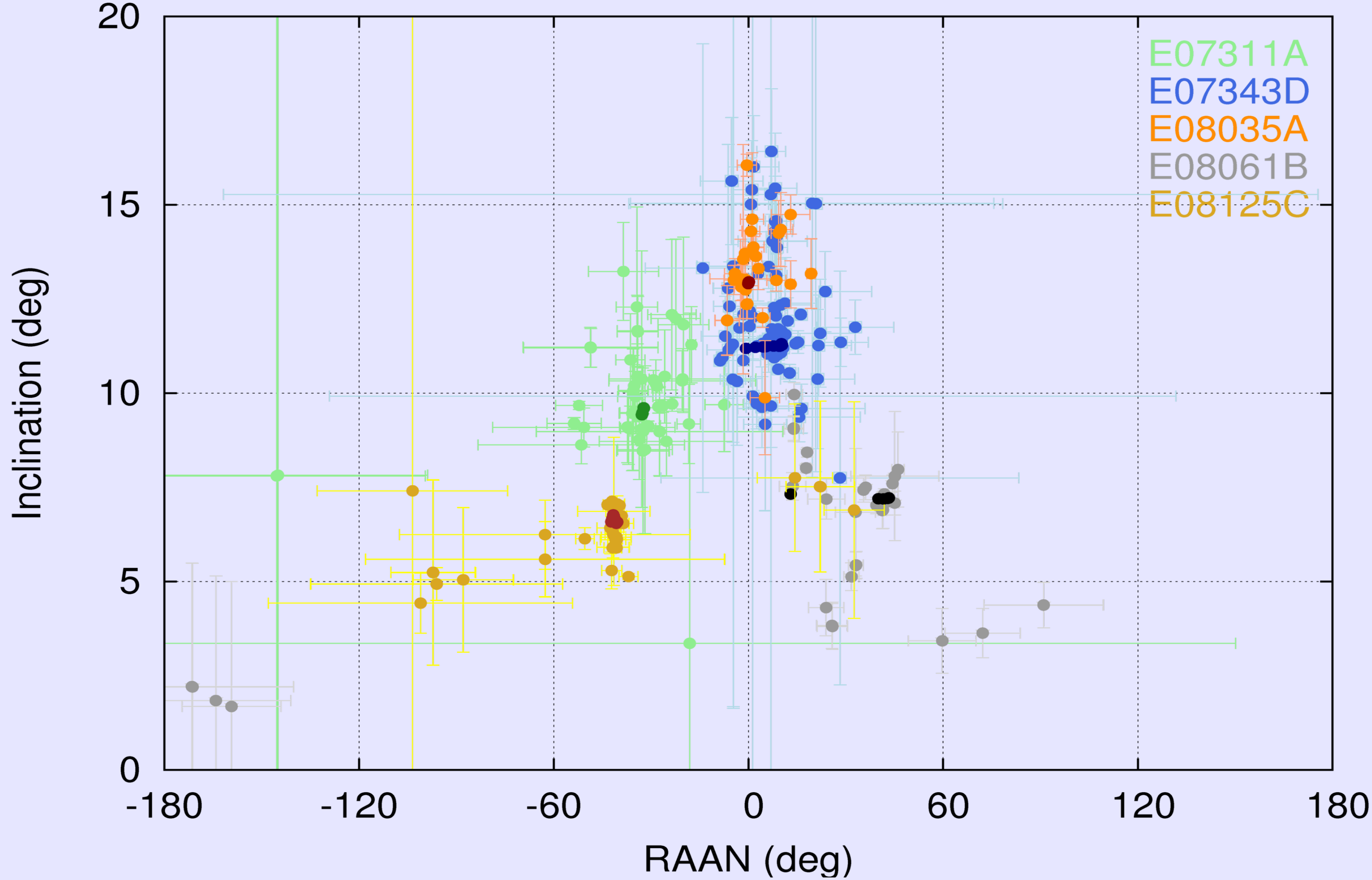
Application to ESA surveys

All test objects



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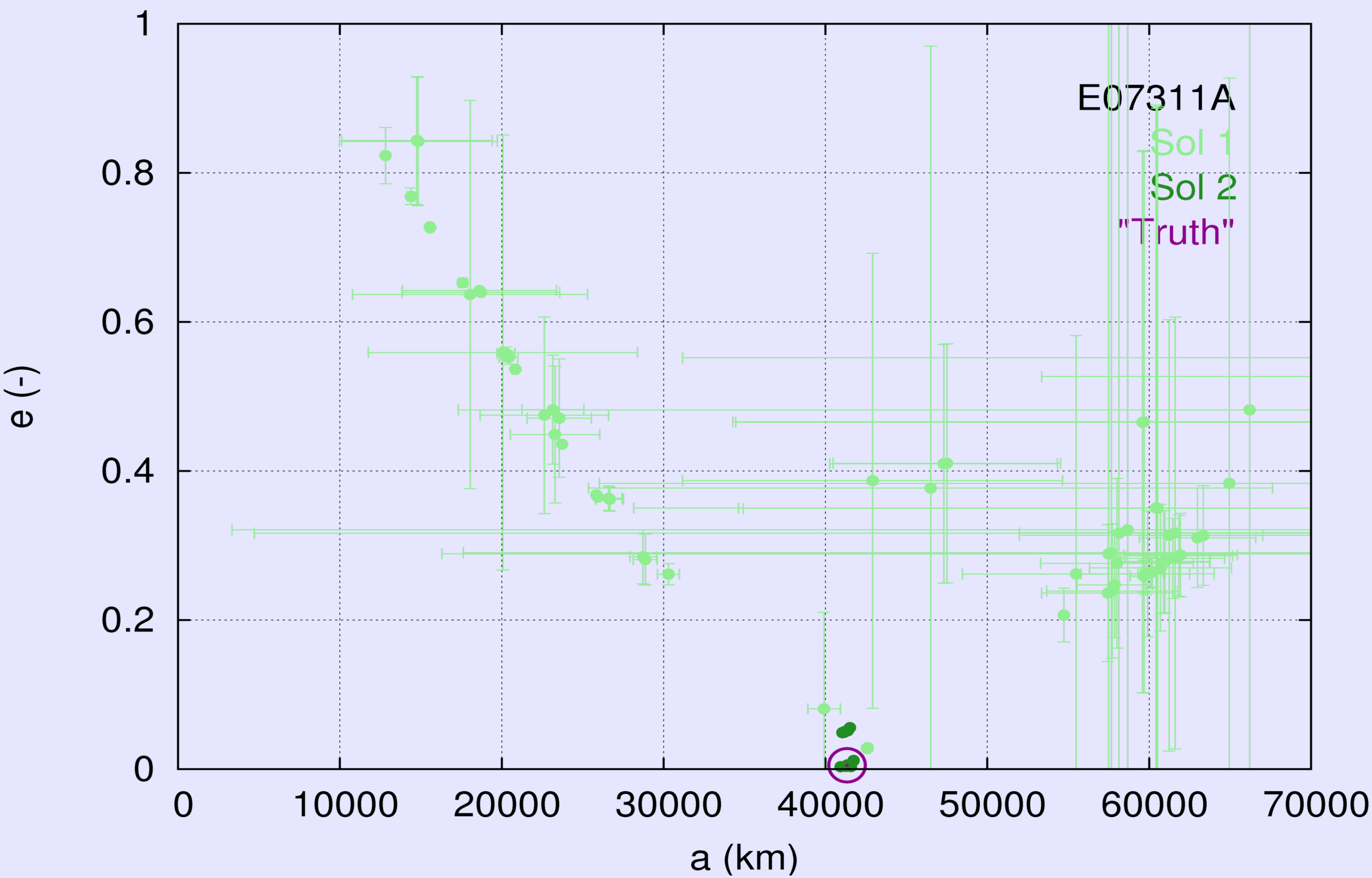
Application to ESA surveys

E07311A



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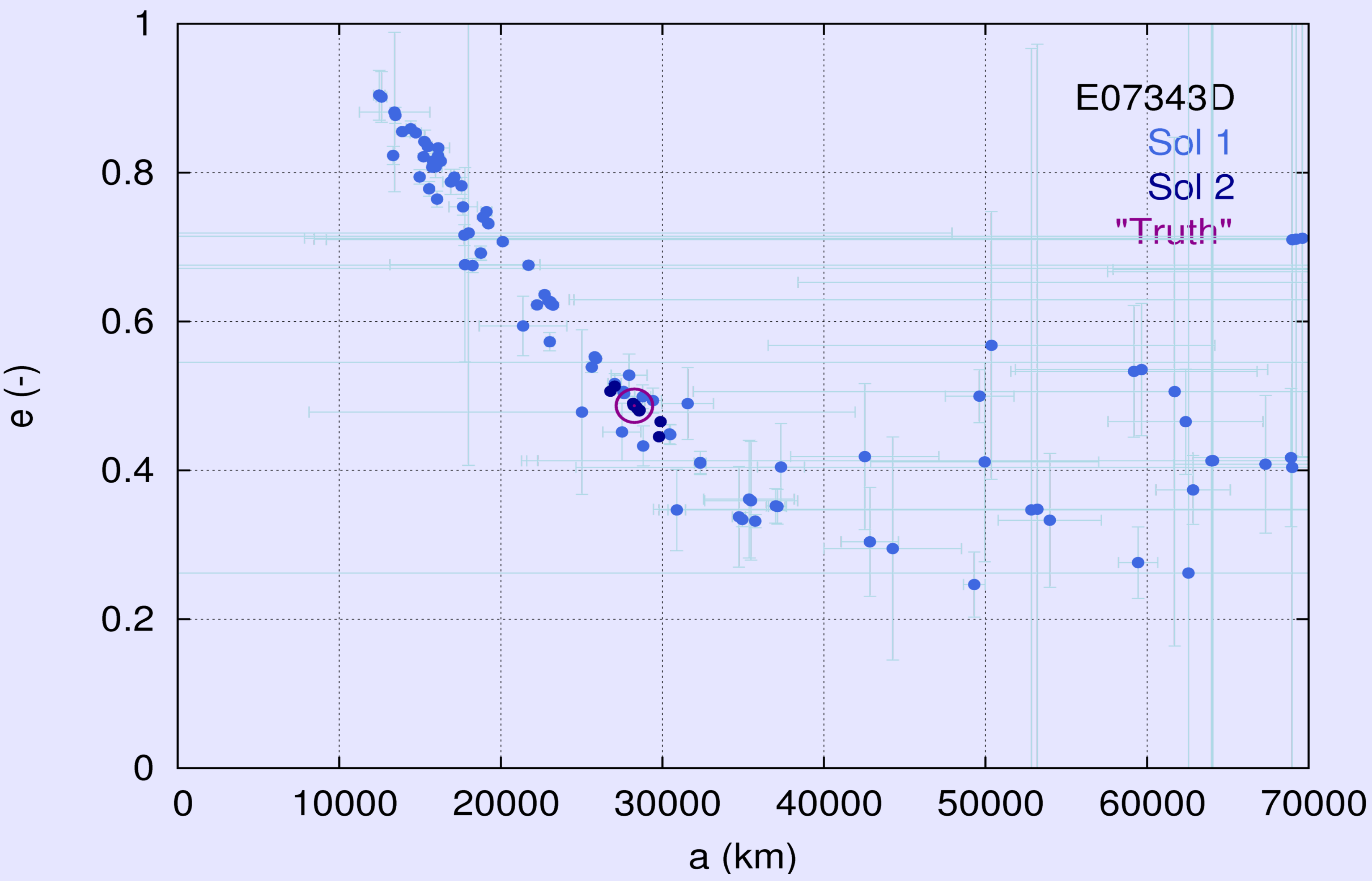
E07343D



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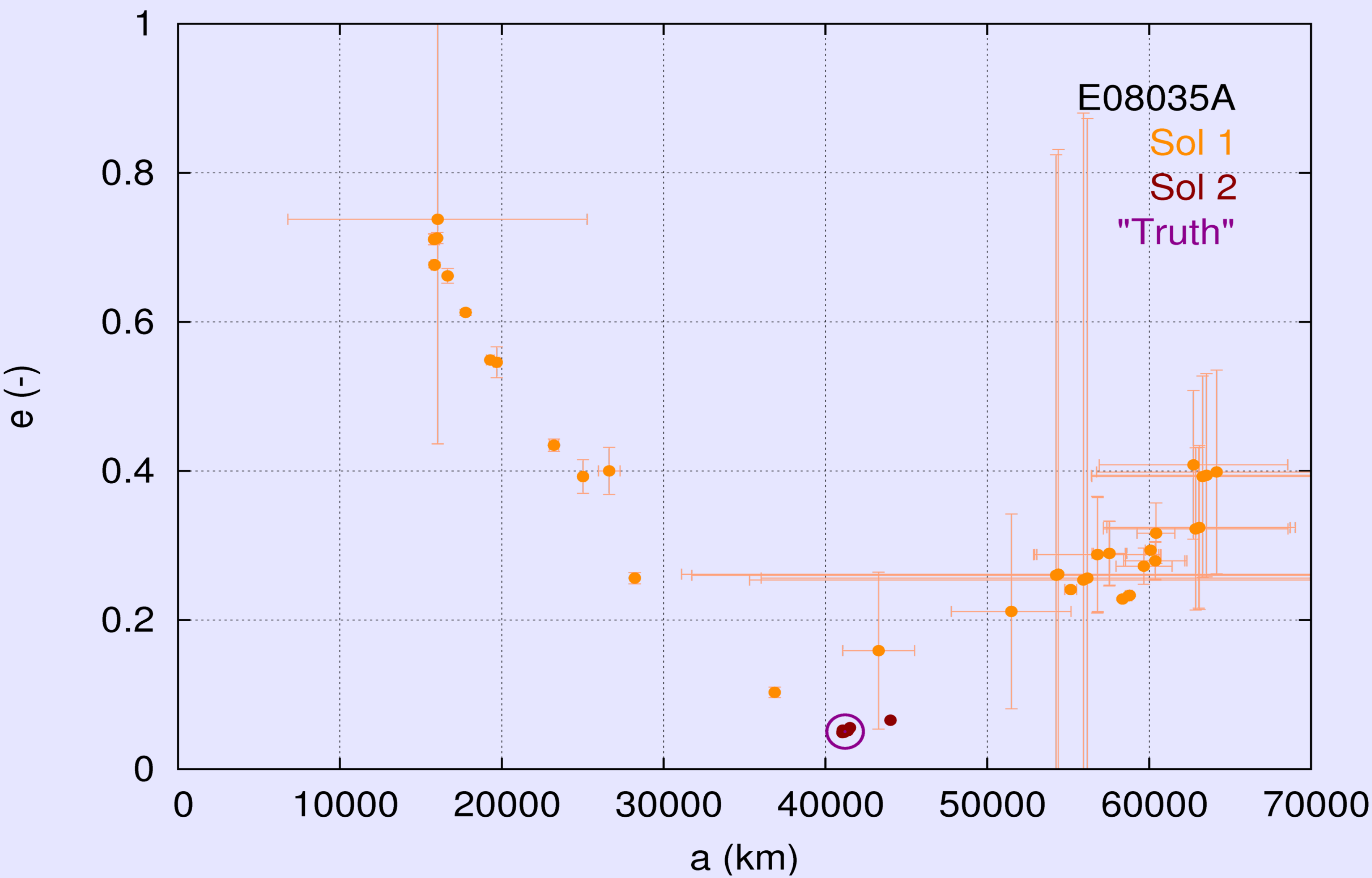

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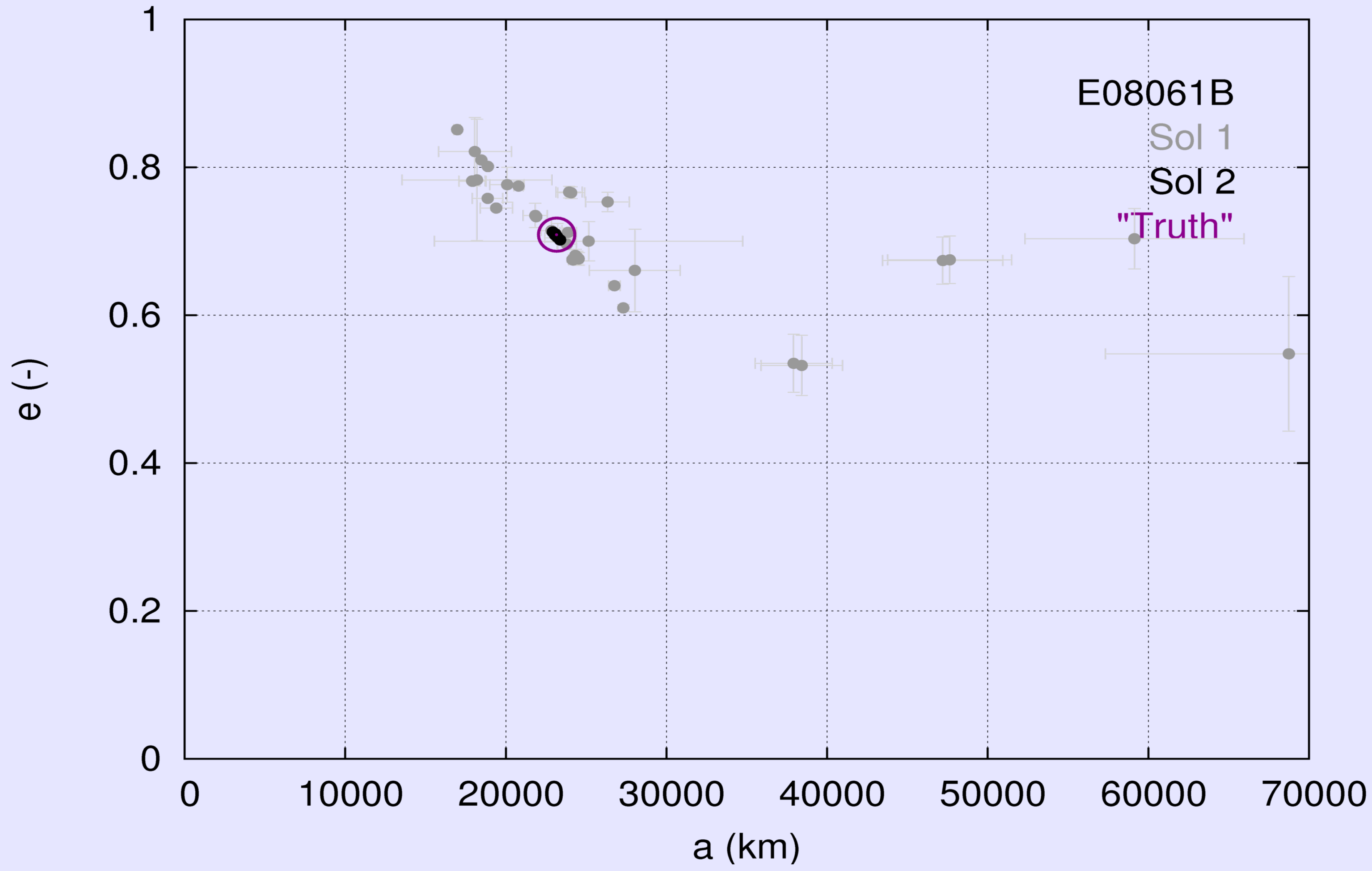
E08061B



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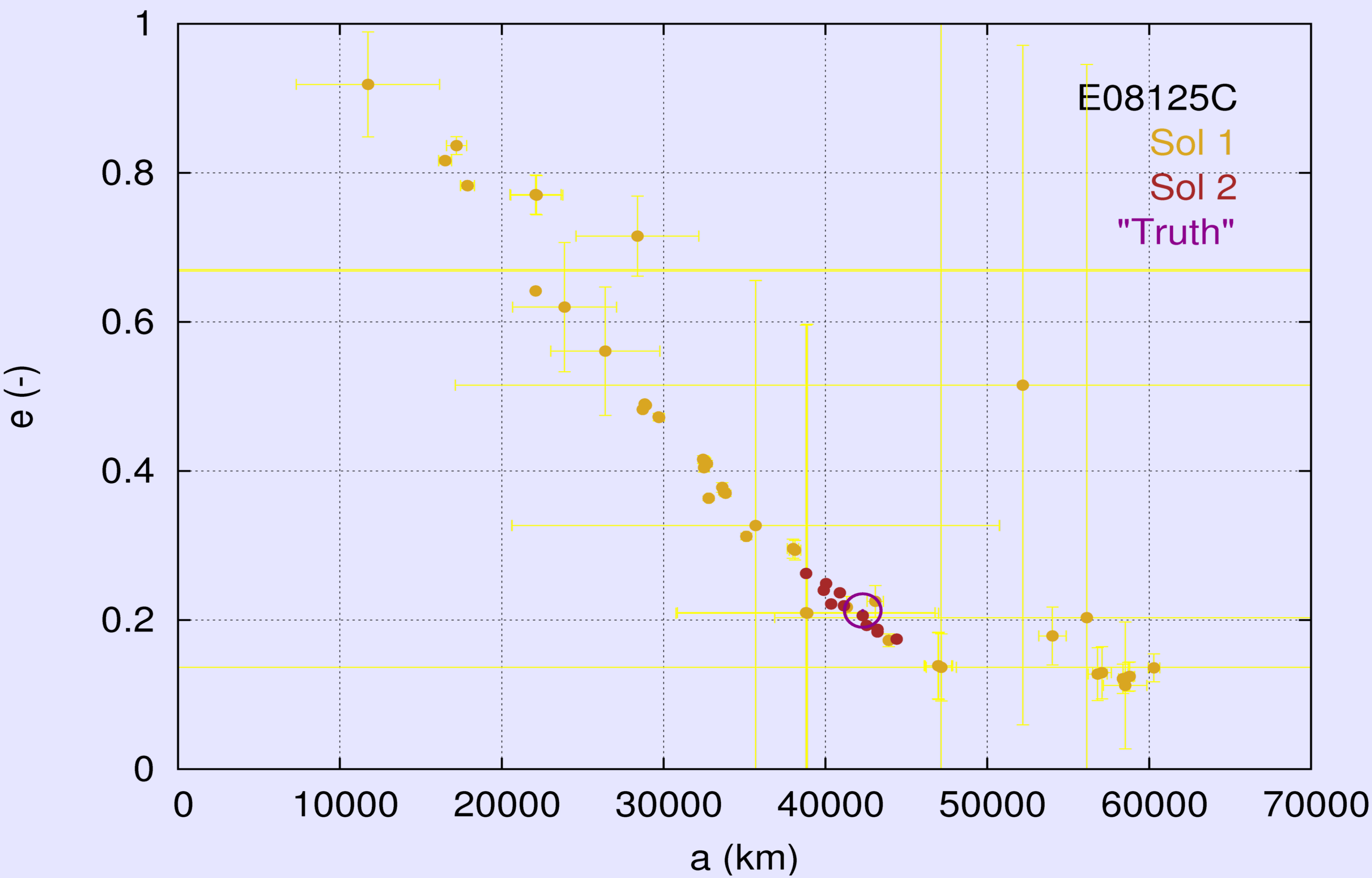
E08125C



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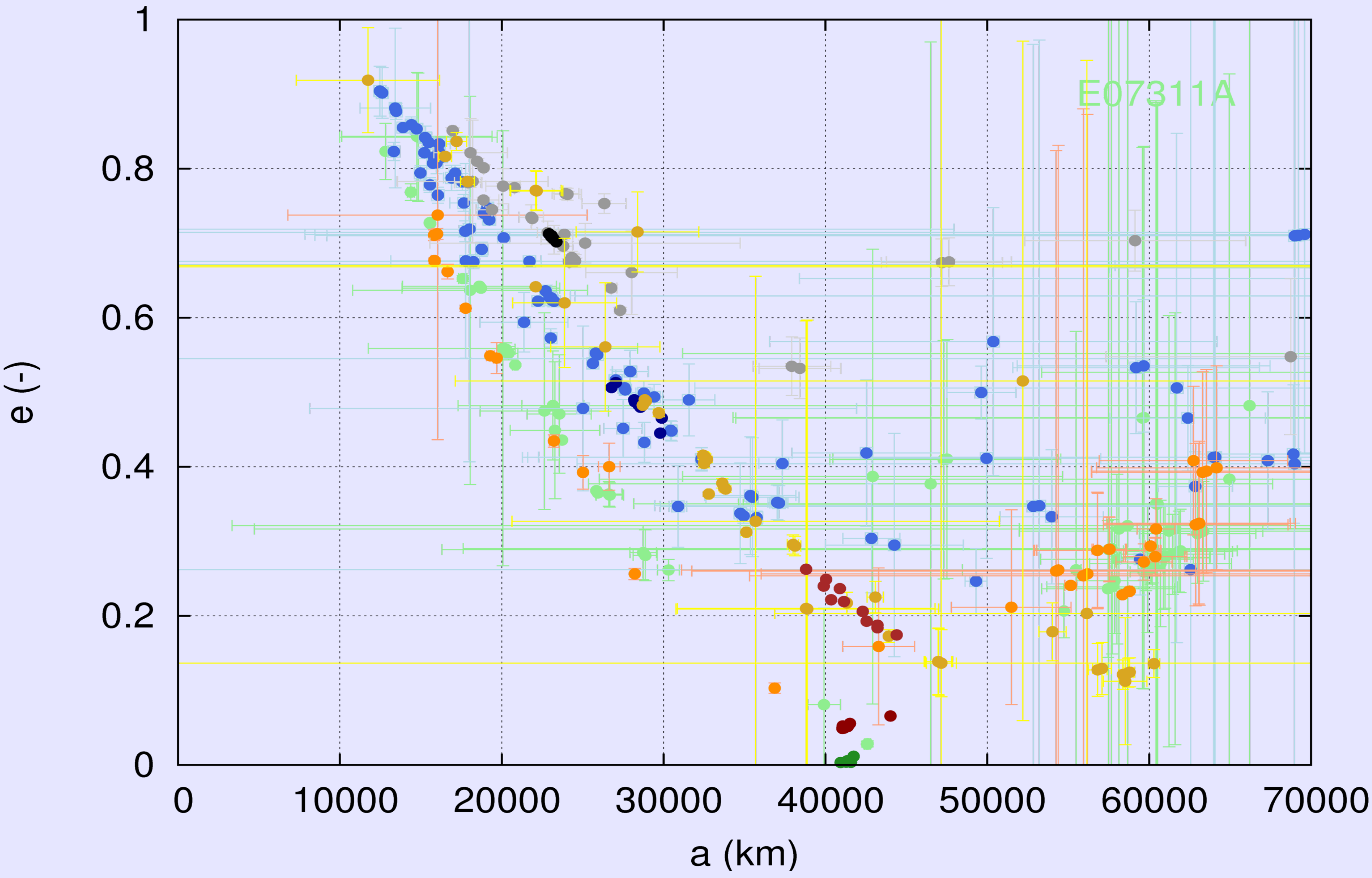

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Outlook

- Inclusion of range observations：
  - Possible，but each range observation adds one line to **A**
  - Optimal (maybe)：use ranges to limit search range in ρ_A and ρ_B
  - How much is the allowed time difference between radar and optical observations?
- Application to real “survey-only” tests
  - Does BNBN2D help in correlation process?
  - Trade-offs with survey-only strategy (pattern) of wide-field sensors

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Summary and Outlook

- Implementation
  - 2D search variant (`BNBN2D`) of boundary value FOD into CelMech
  - Direct interface to ESA's PROOF-tool via plugin-option
- BNBN2D :
  - Extremely flexible tool for various work(s)
  - Easy implementation (on-board?)
  - If $\rho_A \approx \rho_B$: alternative to already employed ground-based FOD
  - Better for eccentric orbits, suitable for space-based observations
  - Typical limits of FOD apply
    - Length of the arclet vs. measurement accuracy :
      (LEO space-based scenario, small FoV survey telescopes)
    - Projection effects” :
      - Pointing in-flight direction
      - Orthogonal to debris orbital plane, ...

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